Τόμος Πρακτικών Φιλοσοφικού Forum «Ανάδρασις»
ISBN: 978-618-82935-0-2

LIGHT, GNOMON AND ARCHAEOASTRONOMY: ON THE SEARCH OF A GNOMONIC PARADIGM FOR ANCIENT CULTURES

Professor Dr Raúl Pérez-Enríquez,
Department of Physics, Sonora University

Abstract

Several years ago, while analyzing the Monument of Stonehenge, we suggested the possible use of a concept now known as the *gnomonic factor*. The evidences we have collected along these years points towards the existence of gnomonic paradigms in ancient cultures. However, few or almost no written evidence appears developing this idea which could be withdrawn from the archaeological evidence: site locations and building orientations, for example. Furthermore, while the knowledge of the gnomon shadows could have been present since the emergence of *homo sapiens sapiens*, when in an erected position he changed his view of space and time through the lecture of his own shadow, gnomon´s presence could be deduced from the use of rods and posts, or from the *menhirs* and *steles*; more complex but at the same time more direct, it could be suggested from the development of calendars and mathematical concepts involved in trigonometry. We can ask: Why there is so few written evidence about the gnomon’s use in ancient cultures?

1. Introduction

ISBN: 978-618-82935-0-2

The year 2015 was declared as the "Year of Light and its applications" by UNESCO (UNESCO, 2015). In this framework, we have introduced from an elementary knowledge of light and its properties, the use of the gnomon as an instrument to produce shadows at the beginning of *homo sapiens sapiens'* development. Through the discovery of the gnomon, men were aware of a concept which could have begun when he got conscience about his own shadows produced by his erected position (Calvino, 2012). The change of the length of the shadow along the day and the yearly cycle of the shortest shadow along the year (between two extreme longitudes) that we now know are measured at the Solstices, became known since: through its implications about the concept of triangles, man has searched for new knowledge and developed what we, now, call ancient cultures.

Today, when we are confronted with ancient monuments like Stonehenge or the Great Pyramid, we have to assume that some of their features came from actual knowledge about the gnomon and its use: The *gnomonic factor* understood as a ratio between the difference of solstices' shadows and the length of the gnomon, could have been used to define the heights of Stonehenge's structures (Sarsen Circle, Trilithons in the Horseshoe array) obeying a calendrical rule (Perez-Enriquez, 2002); or, the use of unitary fractions in Egyptian mathematics, very well documented in several papyri (Gillings, 1982), could have resulted from solar observations using a gnomon. These two examples and many other more could become apparent when an analysis of the possible ways ancient people got that knowledge is done.

Different cultures had diverse forms to approach the same natural events. However, such applications could have arisen from basic ideas coming from, for example, the observation of the Sun. Being (the Sun) a god for many different cultures, the knowledge of its motion in the sky, of its regularity and predictability, could have been the basis for rituals and another kind of activities all of them attributed to astronomers-priests or priests-astronomers. The use of the gnomon as an instrument to project shadows soon became a calendrical tool; the astronomer-priest having this tool could predict when the rain will come or when grains have to be harvested.

The existence of gnomonic paradigms in ancient cultures could then be searched through the study of these manifestations. An important fact that must be taken into account is the few written evidence of the use of the gnomon in the past. There are only a numerable notices of the use of the gnomon: Thales measurement of the height of the Great Pyramid in Egypt; estimate of the circumference of the Earth made by Eratosthenes of Alexandria; Al Batani's principles of trigonometry defined using the shadows of gnomons: vertical and horizontal ones (Stillwell, 2010).

In this paper, we introduce the concept of gnomonic factor through its use in the interpretation of the monument of Stonehenge; then, we present the characteristics of such calendrical idea, suggesting a gnomonic paradigm discovered by G. Hawkins (1963). In section 3, we review few of the known expressions of cultures where the gnomon is taken as a tool. We present, in section 4, the implicit evidence of the use of the gnomon, introduced by Herodotus' History where Dodona Oracle and the

ISBN: 978-618-82935-0-2

Ammon's temple were mentioned and its relation with the platonic gnomonic factor. In the last section, section 5, we make our suggestion about gnomonic paradigms in ancient cultures and invite the reader to remember Heraclitus' quote on the size of the sun.

2. A Stonehenge's Paradigm

The Monument of Stonehenge, located in the Salisbury Plain in England, has been developed since its first stages, along several centuries (Hawkins, 1963). The main feature of its more recent stages is the set of monumental sarsen stones arranged in two structures: the Sarsen Circle and the Trilithons Horseshoe. The other features and stages are not going to be commented here: blue stones structures. The sarsen structures are almost contemporary and their heights seem to be chosen to represent days or more precisely, dates of a calendar.

In Figure 1, a general view of the monument can be seen. Three of the five Trilithons of the Horseshoe and an upright of the Great Trilithon are easily identified. Also, due to the fact that the picture was taken from the south of the monument, a dwarf sarsen stone (S11) can be noted near the centre of one of the trilithons: this half height stone is very important in our analysis as it is apparent in Table 1, below.

ISBN: 978-618-82935-0-2

Figure 1. View of Stonehenge Monument from the south.

In the following table, Table 1, we show the main heights of Stonehenge's structures; in column 3, the ratio of the height over S11's one is shown. In the table, also, two midday sun's shadows are presented: in 1st of January and at the summer solstice.

Table 1. Stonehenge Structures' Heights and Its Ratio w/S11[1]

S11	1st Jan S[2]	SSS[2]	Diff.
2,44 m	8,60 m	1,28 m	7,32 m

Structure[3]	Height (m)	Ratio w/S11
Great Trilithon	7,32	3,00
Altar Stone	5,00	2,05
Sarsen Circle	4,88	2,00
Heel Stone	4,90	2,01

[1] Stone 11 of the Sarsen Circle.
[2] Shadows: 1st January and Summer Solstice.
[3] See Figure 2.

ISBN: 978-618-82935-0-2

Assuming a relation between heights of trilithons and shadows of a gnomon (S11's height would be it), the structures define a kind of calendar shown in Table 2; a calendar where the selected dates would be the summer solstice for the Great Trilithon: days 104 and 259 for the trilithons 1 and 4 as defined for the Sun at half way between zenith and the horizon (45° elevation); and, trilithons 2 and 3 for days 120 and 245, respectively, for the division of the year in three periods of 120 days plus a five day laps.

Table 2. Calendrical Analysis of Stonehenge Structures[1]

Structure Name	Height (m)	Elevation (°)	Shadow (m)	# Day
Altar Stone	-0,10	15.4	8,70	0 – 365
Base Day	0,00	15.8	8,60	10
Stone 11	2,44	26.1	6,16	40
Trilithon 1	6,16	45.0	2,44	104
Trilithon 2	6,55	50.0	2,05	120
Great Trilithon	7,32	62.3	1,28	182
Trilithon 3	6,55	50.0	2,05	245
Trilithon 4	6,16	45.0	2,44	259
Stone 11	2,44	21.6	6,16	324

[1] Gnomon = 2,44 m and Base Day elevation = 15.8°

This analysis was introduced for the first time by me (Perez-Enriquez, 2000) and it is worth value to say that there are only few works oriented to give an interpretation of the trilithons heights. The proposal by North (1996) addresses the point from a more architectural perspective giving lines of sight from the surroundings.

ISBN: 978-618-82935-0-2

The other fact of a gnomonic paradigm at Stonehenge is its location at a place where the Solsticial Lines make an angle of 90° (Hawkins, 1963 and Hoyle, 1986), as it can be seen in our Figure 3. We consider the two solsticial lines defined according to the following: a virtual line going from the summer solstice sunrise towards the winter solstice sunset point; and, a virtual line going from winter solstice sunrise towards the summer solstice sunset point.

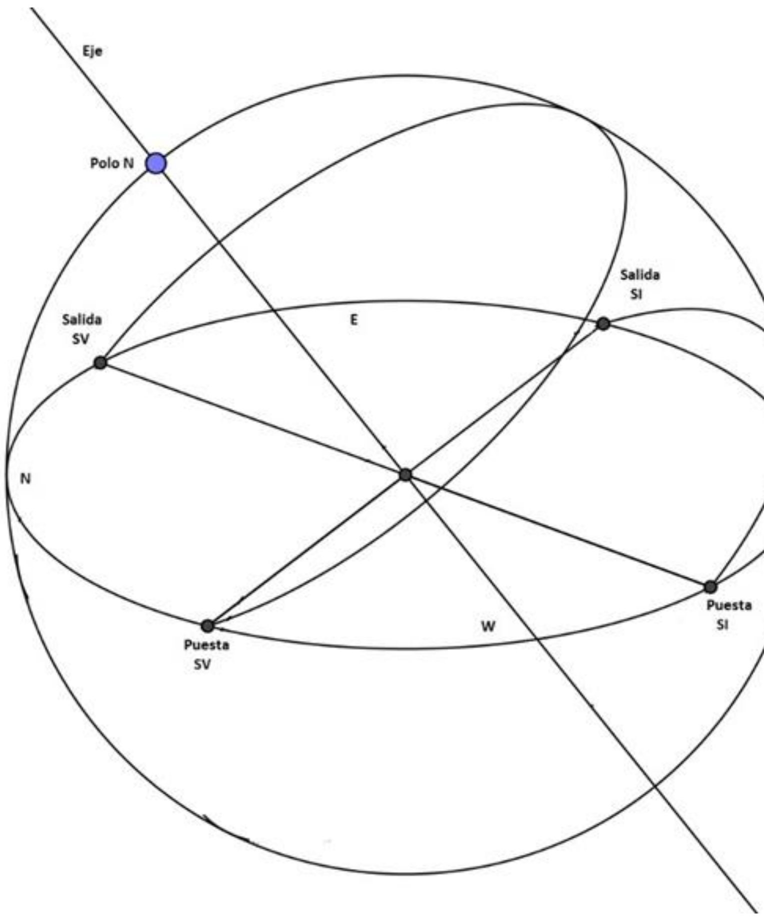

Figure 3. Solsticial Lines at Stonehenge Latitude making a 90° angle.

This special alignment of the solsticial lines appears only in latitudes around Stonehenge and could have been the reason for the selection of the site back to third millennium BC, when the first evidences of

ISBN: 978-618-82935-0-2

occupation appear. Together with the gnomonic behavior of the Trilithon's heights, the solsticial rising Sun could have played an important role during the design phase of the monument.

One can suggest the presence of a gnomonic paradigm well before the use of sundials and the measuring of time, the gnomon being used as an astronomical instrument for calendrical purposes.

3. Other Evidence in Antiquity

The use of the gnomon for other purposes different than sundials became known by man many centuries before; its use as a tool for orientation and seasons reckoning can be found in several ancient cultures. From China to Egypt or from Mesopotamia to Greece, it is possible to identify such applications, many of them not reported or without having written evidence, while others left track of such matters.

3.1 Chinese tradition

One of the most relevant examples of this kind of use of the gnomon comes from China, where in an old document, the *Zhou bi suanjing* (*Zhoubi* for short), there appears the *bi guo* (gnomon ruler) as a tool for determining the 24 *qi* or periods dividing the meteorological year. Back in 540 BC, the lengths of the shadows of an 8 *chi* gnomon (eight feet) observed at solstices were used as it can be seen in Table 3. The *chi* is the unit of length in ancient China (Martzloff, 1997).

Table 3. *Qi* periods and corresponding Shadows in the Zoubi [1]

Name	zhang	chi	Cun	Fen	fen/6
[1] winter solstice	1	3	5	0	0
[2] little cold	1	2	5	0	5
[3] great cold	1	1	5	1	4

ISBN: 978-618-82935-0-2

[4] beginning of spring	1	0	5	2	3
[5] rain waters	0	9	5	3	2
[6] emerging insects	0	8	5	4	1
[7] spring equinox	0	7	5	5	0
[8] clear and bright	0	6	5	5	5
[9] grain rains	0	5	5	6	4
[10] beginning of summer	0	4	5	7	3
[11] grain fills	0	3	5	8	2
[12] grain in ear	0	2	5	9	1
[13] summer solstice	0	1	6	0	0
[14] little heat	0	2	5	9	1
[15] great heat	0	3	5	8	2
[16] beginning of autumn	0	4	5	7	3
[17] limit of heat	0	5	5	6	4
[18] white dew	0	6	5	5	5
[19] autumn equinox	0	7	5	5	0
[20] cold dew	0	8	5	4	1
[21] frost-fall	0	9	5	3	2
[22] beginning of Winter	1	0	5	2	3
[23] little snow	1	1	5	1	4
[24] great snow	1	2	5	0	5
[1] Shadows corresponding to an 8 chi gnomon [2] Taken from (Cullen, 2010)					

Assuming a Winter solstice shadow (WSs) of 13, 5 *chi* and the Summer Solstice shadow (SSs) of 1, 6 *chi*, *Zhoubi* defines the periods appearing in the table (Cullen, 2010). But this supposition implies that the observations were made at an earlier time, about two thousand years before the date assigned to the *Zhoubi* (Li & Sun 2013).

The persistence of the use of such an instrument is confirmed at later times during the Ming Dynasty when the astronomer Gao Shuojing draw the schematics of a gnomon shown in Figure 4. The legends correspond to sun positions and related shadows.

ISBN: 978-618-82935-0-2

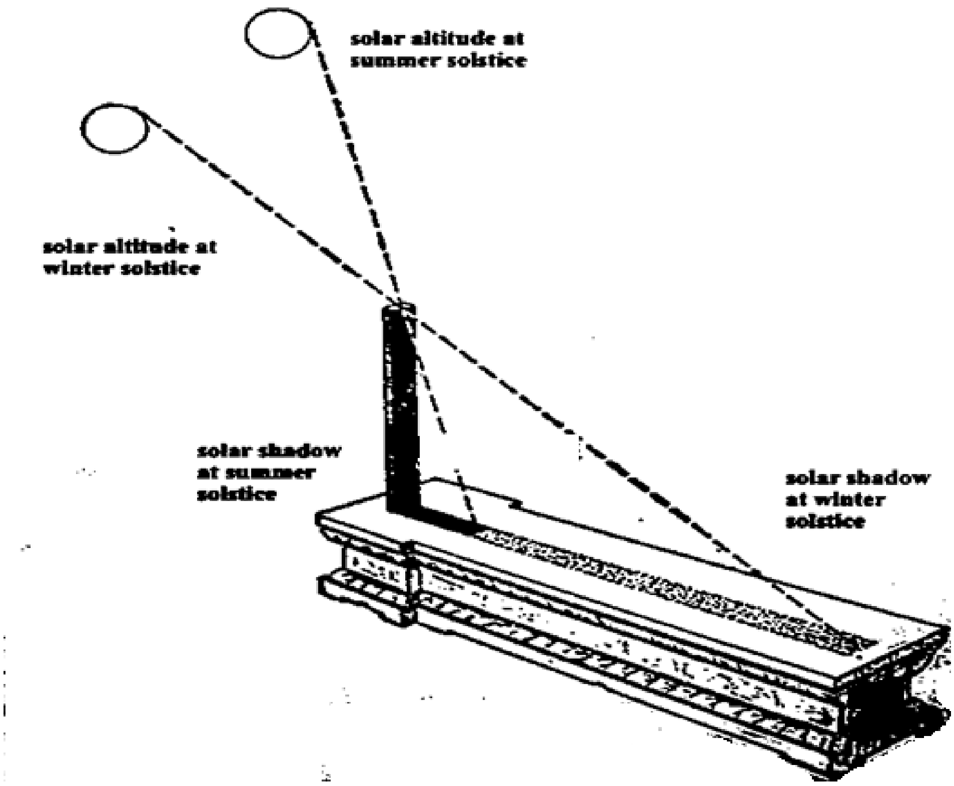

Figure 4. Gao Shoujing gnomon schematics.

It is important to notice that the instrument have had to be oriented correctly, in a North South direction. This kind of *bi guo* was constructed to be fixed to the earth. Many examples of this type can be seen in Beijing today. In this tradition, the Chinese one, a paradigm becomes evident: The division of the year can be registered recording the length of a gnomon shadow between two extreme lengths: the longer at winter solstice and the shorter at summer solstice.

However, the paradigm in the Chinese tradition could be related with the search of measures of the solstice's shadows to the already mentioned values 13, 5 and 1, 6 *chi* of length. This fact would imply that the *bi guo* and, in correspondence, the official residence orientations would have

ISBN: 978-618-82935-0-2

had a non exact North-South value. In Table 4, the values of gnomonic factors for five ancient important residences of China are given with the azimuth in which the specified shadows were observed. The table includes sites of different regions of China. The azimuths were obtained for the Epochs indicated with the aid of the program Stellarium (Chèreau, 2014).

Table 4 Orientation of official residences at selected sites and the *Zhoubi* shadows

Site	Epoch (BC)	Latitude N	Orientation[1]	WSs[2]	SSs[2]	*Fg*[3]
Erlitou	1683	34°.693	192°.22	13.4709	1.5779	1.4866
Luoyi	398	34°.654	167°.77	13.4986	1.5816	1.4896
Yuntang	1493	34°.489	192°.80	13.4987	1.5807	1.4897
Qizhen	1436	34°.484	192°.85	13.4987	1.5560	1.4928
Fengchu	914	34°.424	166°.67	13.4986	1.5491	1.4936

1 Azimuths of the Sun at the direction of the residences (ZAT 2009) and (ATZP 1979).
2 Shadows of an 8 *chi* gnomon found with Stellarium (Chèreau, 2014) at the given azimuth.
3 In the specified orientation.

As it can be seen, all the values correspond to the *Zhoubi* shadows of 13, 5 and 1, 6 *chi* of length, being Erlitou the most extreme one. It is remarkable that the gnomonic factors can be considered with confidence as 1, 5. The deviation from an azimuth of 180° goes as far as 22 degrees East. Then, a gnomonic paradigm for China could be stated as following: orientation of official residences could have been selected in accordance with the measurement of specific shadows at solstices and looking for a gnomonic factor of 1, 5.

3.2 Egyptian tradition

Obelisks can be seen as the result of the development of gnomons in ancient Egypt. At the beginning as a staff they are combined with the knotted rope the *harpedonaptae* (man responsible of the measurement) could have found the proper orientation of the pyramids and temples with great precision (Belmonte, 2012). We cannot be sure of that fact but the very existence of obelisks all around the cities of the old and new kingdoms, suggests that Egyptians were aware of the use of the shadows for many purposes. The tale about Thales measurement of the height of the great pyramid with the help of a gnomon stays as evidence of the kind of knowledge he acquired from his Egyptian teachers and of the enhancement of the mathematics he was capable of.

Again, Thales of Miletus did not use the gnomon for measuring time but he still was confident of waiting the exact moment to make the observation. That moment was the time when the Sun, at its halfway between the horizon and the zenith, revealed its rays: at that moment the gnomon and its shadow were of equal length.

In fact, the observation of the Sun at 45 degree could have been part of an Egyptian paradigm during the search of a location for a necropolis or main building. A study of the platonic gnomonic factor (*fgp* for its name in Spanish – Perez-Enriquez, 2014 – and references in it) provided us with a relation between the *fgp* and the latitude of the important buildings found along the Nile River. In Table 5, the list of sites and the corresponding factors can be seen. It is important to know that the *fgp* is obtained by the ratio between the excess of the Winter solstice shadow to the gnomon length and this same gnomon length.

Table 5 Special Sites in Egypt and Their *fgp*'s

Site	*fgp*[1]
Heracleopolis[2]	1/3
Siwa Oasis[2]	1/3
Meir	1/4

ISBN: 978-618-82935-0-2

Abydos	1/5
Thebes[2]	1/6
Menphis[2]	1/6
Nechen	1/7
Kom Ombo Temple	1/8
Aswan	1/9
[1] Platonic Gnomonic Factor [2] Similar Latitude but different Longitude	

The unit fraction value of the factor just described becomes significant when one takes into account the mathematics developed by Egyptian people: it was based on these unit fractions (Gillings, 1987). The gnomonic paradigm in this case would have been related with mathematics and geometry.

4. Greek Gnomonic Paradigm

Since 450 BC, we find in Herodotus' *History* the idea that with other kinds of knowledge, the use of the gnomon was introduced to Greece by the Egyptians. Anaximander was one of the important philosophers to work with it and to develop concepts and instruments that were later known as *scaphe* and sundials (Couprie et al., 2003). By the year 200 BC, Eratosthenes made its estimations about the circumference of the earth; some authors claim that it could have made two measurements assuming different solar rays' trajectories (Carman & Evance, 2015). We can suggest that the measurement was made with the aid of a gnomon and assuming that the cities of Syene (today's Aswan) and Alexandria were both at the same meridian, he could have used a size of an 1/8 shadow of

ISBN: 978-618-82935-0-2

a gnomon at the Summer Solstice in the second and the fact that the gnomon made no shadow in the former.

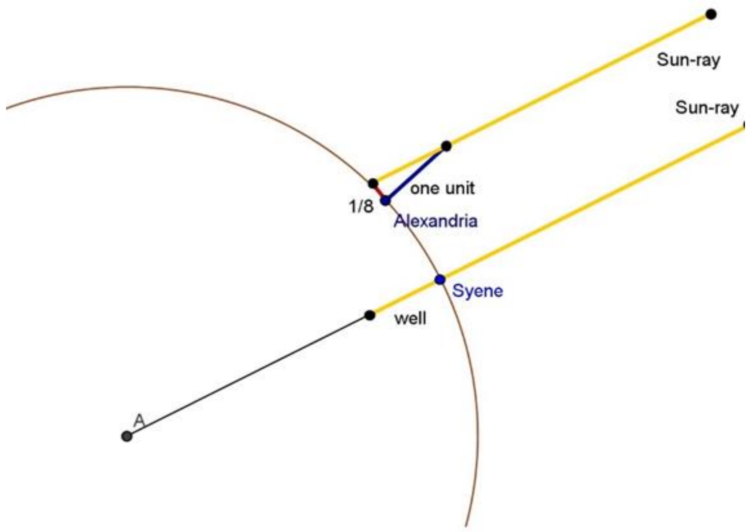

Figure 5. Eratosthenes observations with the gnomon while measuring the circumference of the earth.

He calculated a circumference of 252,000 *stadia* because the distance between the mentioned cities was 5,000 *stadia*: he could have covered the circle with 50 1/2 shadows of one eight length.

Here, in this example, the shadows of a gnomon considered were taken at winter solstice in two different places. Again, the idea of a gnomonic observation takes place and presence while studying natural phenomena.

But the shadow at winter solstice combined with that observed when the Sun was at 45 degrees, as was the case of the Egyptian tradition for the location of sites, appears, also, in Greek tradition: Herodotus, in 'Euterpe', says:

'The following tale is commonly told in Egypt concerning the oracle of Dodona in Greece, and that of Ammon in Libya. My informants, on the

ISBN: 978-618-82935-0-2

point, were the priests of Jupiter at Thebes. They said "that two of the sacred women were once carried off from Thebes by the Phoenicians, and that the story went that one of them was sold into Libya, and the other into Greece, and these women were the first founders of the oracles in the two countries."' (Herodotus 450 BC)

Yes, the ratio between the length of the shadow at winter solstice minus that when the Sun was at 45°, and the gnomon's length, had the value of 1 (one) and one third (1/3) for both sites, respectively. The important of these unit fractions factors (1=1/1) become apparent when the gnomonic triangles described by these two values are analyzed:

i) The 1/1 case implies a triangle with sides 1:2:root(5) also known as the Platonic Triangle because it was described by Plato in Timaeus (Artmann & Schäfer, 1993);

ii) The 1/3 case conducts to a triangle which sides are 3:4:5 also known as Pythagorean Triangle; a triangle well known in ancient Egypt (Rossi, 2004).

The main two oracles mentioned by Herodotus' were placed at sites where the specified shadows made these very important geometric figures. One is obligated to ask: did the priestesses were looking for those places?

In a recent study, which will be published soon, we have found other factors involved in the construction of Apollo Temples, for example, that bring the issue of a possible gnomonic paradigm. For the moment, the Greek paradigm could be stated as it follows: The length of the shadow at

ISBN: 978-618-82935-0-2

winter solstice while compared with the length of the gnomon could be used as criteria for an oracle location.

5. Final Remarks

Several ancient traditions from different parts of the world have been reviewed with special interest of the possible observation of the Sun; in particular, its observation through the shadow casted by a gnomon (a vertical gnomon). It has been detected a main idea about this kind of measurements between the cultures here considered. Other cultures have similar uses of the shadows of a gnomon. Not in all of them the implicit paradigm is the same: while for some seems to involve the shadows at solstices; for others are the shadows at the equinoxes. Most of them involve the winter solstice shadows because of the latitudes of their zones of influence.

The presence of these uses of shadows does not appear as explicit as one could suggests; the only obvious case is that of the shadows reported in the *Zhou bi suanjing*, where their values are reported for a well-defined gnomon (one with eight *chi*). In such a case, the orientation of the bi guo would be different from North-South direction.

The case of Stonehenge tradition, Wessex culture, if confirmed with other indicators, would establish a role of the paradigm in the architecture. The combination of an extreme shadow with those of other dates along the year, including the Summer solstice, makes this tradition a very significant one for the definition of the paradigm.

Maybe, the reason for the infrequent mention of the gnomon in the ancient literature is based on the fact that since the beginning of civilization or evolution of man, the shadows of himself, its staff or posts, were present all the time, so that they were given of granted.

Finally, let us remember a quote by the great Greek philosopher Heraclitus: "[Concerning the size of the sun: it is] the width of a human foot". This quote acquires relevance to the topics reviewed along this paper because it may describe the Sun observed with an instrumented gnomon: a gnomon with a pinhole at its end. The image of the Sun at Winter Solstice would have been one foot wide as it is precisely at Stonehenge (Perez-Enriquez, 2013).

References

Archaeological Team of Zhou Plain (1979), *A Preliminary Report of Western Zhou Building Foundation at Fengchucun*, Qishan, Shaanxi. Wenwu 10, pp. 27 – 37.

B. Artmann and L. Schäfer (1993), "On Plato's 'Fairest Triangles'" (Timaeus 54a), *Historia Mathematica*, v. 20, pp. 255 – 364.

J. A. Belmonte (2012), *Pirámides, Templos y Estrellas*, Crítica.

R. Calvino (2012), *Modelo Antropognomónico de la Evolución Humana*. Internet. (Available at: http://planetaantropologico.com/modelo-antropognomonico-de-la-evoluciin-humana/)

Chr. C. Carman and J. Evans (2015), "The Two Earths of Eratosthenes", *Isis*, 106:1–16.

F. Chèreau et al. (2014), *Stellarium*.

D. L. Couprie, R. Hahn and G. Naddaf (2003), *Anaximander in Context*, State University of New York Press.

Chr. Cullen (2010), *Astronomy and Mathematics in Ancient China: The 'Zhou Bi Suan Jing'*, Cambridge Books Online.

G. Hawkins, (1965), *Stonehenge Decoded*, Souvenir Press.

Herodotus (450 BC), Book II, Euterpe, *History*.

Fr. Hoyle (1986), *De Stonehenge a la cosmología contemporánea*, Alianza Editorial.

Li, Yong and Xiao-Chun Sun (2009), "Gnomon shadow lengths recorded in the Zhoubi Suanjing: The earliest meridian observations in China?", *Research in Astronomy and Astrophysics*, v. 9, pp. 1377 – 1386.

J. Cl. Martzloff (1997), *A History of Chinese Mathematics*, Springer-Verlag, pp. 26-29.

J. North (1996), *Stonehenge: A New Interpretation of Prehistoric Man and the Cosmos*.

R. Perez-Enriquez (2013), "On the use of 'Pinhole Optics' in ancient times", *RAO* XXIV.

________________ (2014), "Plato's Triangle and Gnomonic Factor: An Application to Herodotus' Oracles", *Mediterranean Archaeology and Archaeometry* Vol. 14, No 3, pp. 45-53.

C. Rossi, (2004), *Architecture and Mathematics in Ancient Egypt*, Cambridge University Press.

J. Stillwell, (2010), *Mathematics and Its History*, Springer.

Zhouyuan Archaeology Team (2002), "Brief Report for the Years 1990-2000 of the Excavations in Western Zhou Building Foundations at Yutang and Qizhen in Fufeng County Shaaxi Province", *Archaeology* 2002, issue 9, pp. 771 - 794.