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THE EPISTEMOLOGICAL IMPORTANCE OF MUSIC
Η ΕΠΙΣΤΗΜΟΛΟΓΙΚΗ ΣΗΜΑΣΙΑ ΤΗΣ ΜΟΥΣΙΚΗΣ

Konstantinos G. Papageorgiou
MSc, cand. PhD
Department of Philosophy, Pedagogics & Psychology, National & Kapodistrian University of Athens
konpapag@ppp.uoa.gr

Here we must address a major epistemological issue regarding episteme, its very foundations and its deeper qualities. As though engineers planning to add a new part to some machine, we must be knowledgeable enough of our own "machine", i.e. science, of its ins and outs, of its specifications as well as its limits.

First things first, we should make it clear that science is *not episteme* (επιστήμη), which, the latter, is *not* mere "knowledge" (<*scientia*), but a "logically structured system of knowledge". Science is in a certain sense the opposite of episteme, as we have already argued elsewhere (Παπαγεωργίου & Λέκκας, 2014). Episteme, incarnating the axiomatic system of mathematics, is based on broad surveillance, which results from experience after the processes of analysis-synthesis and abstraction-structure -the two basic methodologies. On the other hand, science is based on experience, and is prone to *the fraud of the*

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senses, a fault it regularly commits. Physics is the main representative of science, and statistics, an otherwise valid mathematical field, its main tool for making sense of the world. The problem with statistics is the way it is used, i.e. instead of logic.

More often than we should, we focus on a particular tree rather than on the whole forest. In the case of *science*, it is commonplace for someone to consider oneself as having become some sort of sage by means of investigating, even thoroughly, no more than one isolated tree -a practice wholeheartedly supported by *academia*. That is a common mistake when knowledge is approached from the point of view of experience and not from the indeed vantage point of broad surveillance. This is another way to forget that episteme (or science if you' d like) is the best way to serve our pre-theoretical intuitions, which are always stronger than any of their theoretical counterparts; so strong indeed, that, when they collide -a common instance in logic-, pre-theoretical intuitions triumphantly prevail over theoretical structures, even elaborate ones. This is the main reason we view theoretical approaches leading to paradoxes as problematic.

When I refer to pre-theoretical intuition, I am not expressing the belief that there is a ready-made set of knowledge waiting to be expressed through specific areas of mathematics, but rather, a strong inclination to accept e.g. A = A as inherently true and its counterpart, a contradiction, as inherently false. However, efforts to rely too much on such intuitions are a slippery slope leading to relativism and even to *scientific empiricism*, setting fire on the very foundations of episteme. A more suitable approach should be this:

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considering episteme as a path of self-improvement[1], basic intuitions should be quickly rejected and replaced with more sophisticated and elaborate ones. This should be achieved through dwelling upon the subject of mathematics. Should we accept that improving our knowledge, together with the quality of that knowledge, is not possible, then, at least as far as I am concerned, the feeling of futility rises.
Being sane enough as engineers are, we would not -and indeed should not- expect our machine, i.e. *our* creation, to do things such as:

- dictate us its own reason of existence,
- have as an output our own input, or
- replace our initial input with its output.

However, some more knowledge is required before we became qualified technicians, *hoping* to manipulate our machine's inner workings. Two things are essential: what we wish to achieve and the knowledge of how to achieve it. Let us now make this analogy explicit and examine how the previous apply to episteme.
A physics student didn't like the idea that while mathematics is an essential part of physics, physics is by no means an essential part of mathematics. His argument was that it is merely a coincidence that physics utilizes mathematics to such an extent, and, things *could have been different*. Well, yes. But since things are not different, unfortunately for him, due respect

[1] Being in the state of knowing is *de facto* considered to be *better* than the state of not-knowing.

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must be paid to mathematics. However, is that really so? Could things have been any different? Not really. Anyone wondering about such questions has misunderstood what mathematics is. Mathematics represents the current evolutionary stage of our substantial characteristic as humans, i.e. *abstract thought*. In their vertebrate stage, humans started out as fish, then continued as apes and so on and so forth. The last few millennia, we have been able to think increasingly *in abstracto*. Our experiences have been transformed into a progressively broader surveillance which has brought about more and more symbolic thought. The climax of such a symbolic thought is typified in mathematics. The latter is not just another set of useful tools, as some of our physicists friends would like to think. Representing the best qualities of our abstract thought, maths has righteously become *the* matrix towards articulating methodology and deeper structure in every other field. It is necessary to understand that mathematics is pretty much like an engine. We tell it what to produce, and do not -or *should* not- expect it to merely start running on its own producing perplexed results which would be significant for us, exactly *because of* their complexity; nor should we consider it trustworthy or valuable (or non-trustworthy or non-valuable), just because along the way, some of its results happened to have become useful. A machine running wild and doing or producing some useful work accidentally is not a desired one. Mind you, however: when I refer to the production of work, I refrain from explicitly mentioning any application; mathematics is not made for any purpose other than producing a coherent self-contained body of knowledge. As a matter of fact, treating

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mathematics only as means towards specific ends -as e.g. physicists or computer scientists do- is a practice incompatible to the statutory conditions of mathematics: the natural evolution of thought is climbing towards higher levels of abstraction. Regressing back to the level of give-and-take with reality is merely a relapse, a sort of backslide. Mathematics may not be goal-oriented, pretty much as is the trend nowadays (*cf*. developing logic *for* computer science) but rather, specifications-dependent; in other words, telic cause may not predominate over necessary cause -quite the opposite. Developing only *some* areas in mathematics, specifically areas we consider more valuable because of their applicability, is a very bad way of constructing a consistent and thorough theoretical structure. Inevitable deficiencies will create a fault that may simply collapse at some point under our own feet.

These words of caution should be enough to warn us that it is not at all fundamental, significant or even legitimate to examine the success of the reintroduction of music to mathematics, *a priori* (before we created it), or *a posteriori* (after we have produced it), based on its effects on e.g. physics or economics. Moreover, being biased when articulating a theory, sets limits to our understanding of the theory, its versatility and its interpretability. Notwithstanding the preceding statements, at least to some extent, we shall evaluate here some of the outcomes of the great (and completed) project of reintroducing music to mathematics; not because we should, but merely because we are able to.

Based on that, we must be able to know why we need mathematics, what mathematics does, and what it will do if we add a part to it -or how basic this part is in relation to the whole. When forgetting the specifications we have chosen for the system of mathematics, while going on treating mathematics as a self-contained entity, i.e., without *us*, and injecting it with our pre-theoretical intuitions, expecting from it to produce as output our own specifications, we shall in fact be treating an open system as a black box and a single tree as a whole forest.

We should be able to foretell what is going to happen to the system, and afterwards, had we been good craftsmen, things should go as planned. This is what I am going to do here: first predict, and then, taking advantage of our experience, go on to see how close our predictions were.

What is music? Music may be defined as the art utilizing pitches and rhythms as its material. What is mathematics? Mathematics is the abstract archetypal space where the relationships of void symbols are deployed, defined and developed.

What does mathematics include? Subdividing mathematics into fields might be more challenging than it appears to be at first glance. A classic initial categorization from the old times, despite its various problems, is the *quadrivium*.

In the *quadrivium* there are two couples:

I. arithmetic and music,

II. geometry and astronomy.

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Various metaphysical beliefs have plagued the *quadrivium*, even in the era of Pythagoras; lack of written testimonies on behalf of Pythagoras only made things worse.

Logic is absent from this quadruple scheme, but only seemingly: it is presupposed and indeed, it is an inherent part of the whole system. In contemporary mathematics, several extra fields have been added, others echoing this classic *quadrivium* more, and others less, or not at all. What is important is not to merely enumerate current fields of mathematics and present a list; ideally, one should find a systemically sound process for distinguishing among various fields based on the decoding of systemic core assumptions, specifications and functions. However, this is beyond the scope of this presentation. Suffice it to say that music "incarnates" arithmetic, as well as its modern expressions, i.e. combinatorics, number theory, algebra etc.

Is music rightfully a discrete field of mathematics? The criterion for judging that is pretty straightforward in mathematics. What is the axiomatic system of music? Music may be viewed as a special case of number theory. There already exists the branch of mathematics called *harmonic analysis* from the Greek word *αρμονικός*, meaning *consonant*, *pleasant*, or even *skilled in music*. It deals with wave-functions, as well as the generalization of the concepts of Fourier series and Fourier transforms. Fourier analysis, in specific, deals with periodicity. Its foundations lie in trigonometry, being itself substantially different from the latter in order to be conceived as a different branch, as much as the theory of music, based

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itself on Fourier analysis, may be perceived as sufficiently different from both trigonometry and Fourier analysis. Different, but relevant: the theory of music, being an elaborate examination of periodicity expressed as relations among musical intervals, i.e. rational numbers, is expected to provide tools for examining the same fields of application as trigonometry and Fourier analysis do. Later on, we shall see some examples. But, right now, let us consider this: in what way is the theory of music different from its counterparts, i.e. what are its substantial characteristics?

What is a triangle? It is a shape and thence a figure -of course. Real triangles exist in our world; in nature or made-up from us. When I was in high school, our mathematician always carried a big triangle with him in order to sketch still other triangles on the blackboard. Are triangles part of the real world then? Does geometry lose its fully abstract nature because one may see and touch *things* with triangular shapes? Nobody should accept such an assumption. However, there still lurks a fundamental antithesis between the way science and episteme conceive this concept: for episteme, a real triangle resembles the mathematical archetype, whereas for science, an abstract triangle resembles a real triangle, such as the one my old teacher used.

The mere fact that real and imaginary triangles are visual perceptions or conceptions respectively, does not make geometry any less abstract. The same holds for other forms of surveillance. Anything that may be accessed through senses such as hearing and touch need not be any *less* abstract.

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This is what sounds are: sure, there are musical works we listen to, as well as musical scores, violins and flutes, the same way there are triangular arcs in architecture, or triangles on a painting. In exactly the same fashion that abstract triangles exist as abstract representations, sounds, i.e. musical intervals, may be said to exist as both abstract representations and parts of our world experience. Both concepts (triangles and musical intervals) pre-exist as absolutely abstract in mathematics, i.e. with no real content or meaning whatsoever, but open to any kind of plausible interpretation.

Now, the utilization of such abstract concepts -triangles, circles etc.- has proved to be very helpful in a series of applications. In mathematics proper, the existence of whole fields has been made possible -fields such as trigonometry and vector analysis. The sciences, i.e. physics, economics, chemistry, etc., have been extensively employing triangles or other mathematical fields using triangles. Exactly because what I state is so obvious, by means of analogy, one should have by now acquired a basic understanding of what a similar abstract musical theory may look like or how it works -at least in principle.

Imagine now trigonometry without triangles or trigonometry made with other figures, such as squares or octagons. Its very name would be different, since triangles would not have existed ("tetragonometry" or "octagonometry"). Would the use of certain other figures produce *more* or *less* similar results to what we now know as "trigonometry"? Perhaps yes perhaps not. Someone who somehow knew trigonometry would easily discern that what we were doing in this hypothetical situation would merely

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be an *Ersatz* trigonometry and that we were in desperate need of, well, triangles. As in the paradigm of the Ptolemaic universe, we would get models to describe reality that would have been needlessly perplexed.

The same applies to music. In much the same way as triangles would revolutionize "tetragonometry", music revolutionizes harmonic analysis, and, even points towards beneficial changes of still other mathematical fields. Three-storey fractions, instead of the classic two-storey ones, are a simple example taken from the easy-to-handle functional fact that truthfully generalized musical chordal harmony is at least triadic. The need that the set of the rational numbers be reducible also stems from the way we enumerate rational numbers in music: rational numbers always expressing specific musical intervals.

The theory of music has already been sideways and provisionally applied to astronomy by D. Lekkas (Lekkas, 2001). Without intending to delve into the specifics of this contribution here, I shall briefly present the main points, while dear readers are referred to the original article. The deep underlying structure of the disc accreted into the known solar system may be reconstructed in terms of harmonic analysis and synthesis, much in the way scoring works in music. One may successfully use the mathematical theory of music to model gaps and clusterings of matter *mutatis mutandis* as consonances and dissonances of how factual musical intervals behave. Modelling celestial systems as "*fluid cymbals*" or pairing planets based on their musical-geometrical characteristics is a major accomplishment derived from the musical theory of mathematics. Yet it

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is *not* about physics. Physics, when and if done properly, may produce the laws governing such instances. This is not at a major concern for mathematics; for maths, an ellipse is not a figure resembling a planetary orbit; on the contrary, an orbit resembles an ellipse. Ellipse is an abstract archetype, whereon one may decide to fit, match or assign data observed in physical phenomena. The decision may be based on a working hypothesis, i.e. a hypothesis *not* in need of any supporting argument; support comes later and only in the form of strengthening our hypothesis -never of verifying it. The same way we may assign astronomical phenomena to mathematical archetypes. For Kepler, and for us, it was sufficient to express the orbits of the planets as three simple mathematical equations. The same principle applies to the structure of our solar system when we express its form and movement as musically derived algebraic geometric equations. The same also applies when explaining or predicting the prevalent existence of twin planets, or the form of galaxies as fluid cymbals -or even when one resolves complex sets of planetary orbits, backwards, on the musical staff, in order to calculate the initial structure of the proto-disc of a single-star solar system such as ours in the remote past of its initial stages.

While astronomy is the usual suspect in regard to "music and mathematics" (*cf*. "Harmonies of the Spheres"), our third example may be an unexpected one. The theory of music has produced major advances in the field of cultural theory and anthropology. How? Musical systems up until now were routinely recorded -and just that: a typical empiristic

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approach. The fourth part of D. Lekkas's dissertation includes the production of a plethora of musical systems *a priori* (musical systems based on surveillance). The procedure includes categorization of rational numbers according to certain criteria and properties (ascending intervals, octave-reduction, prime factor analysis etc.). This original and purely systemic production of the musical systems has created a hierarchical map of such systems across the globe, diachronically and trans-culturally. Then the same methodology has become ready to be used as a diagnostic and analytical tool regarding what has actually happened in human cultures. Before even the simplest musical work of any culture may be heard, anywhere in the world and at any-time[2], one may be pretty sure about matters seemingly irrelevant, such as social structure, general religious beliefs and military aggressiveness, to name a few. All these just by knowing, *a priori*, the frame of mind that is needed to produce the concerned musical system, i.e. the musical scales, or maybe even to take notice of the way the civilization under examination constructed its musical instruments.

The two primeval musical systems in actual practice, as found crystallized on the piercings of archaeological finds of flutes and pipes, are but two, and that for good reason, thoroughly analyzable by mathematics: a pentatonic and an heptatonic one; everything else is systemically produced by these two, each coming in simple dual or triple evolutions, transformations and

[2] Or even irrespective of its availability, based on the discovery of even a single musical scale

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corrections of the two mentioned. The spectacular fact is that the birth and spread of these two scale archetypes is anti-theoretical in its conception, but diagnosable and curable through pure theory. Would this be a factual demonstration of the imperfection of the human being, its tendency to create inherent visions of reality in a reflected, upside-down and sensory fraudulent fashion? There is plenty to follow, but it is almost certain that the Pythagoreans would be enthusiastic in support of this statement and take it as a powerful argument in their own ever-burning fire in their spiritual altar of the all-abstract archetypal mind of the universe and of the spontaneous tendency of the human animal to misinterpret it and of the thinker's duty to set the whole thong straight.

Episteme as culminating in pure mathematics is conscious of the fraud of appearances (*phaenesthai*) to the true essence (*einai*), which is spiritual. Science, on the other hand, essentially empirical, has promoted *phaenesthai* into the only reliable *einai*, thus overturning and erasing the very idea mathematics has been created upon; therefore, in a very true sense, science is an intellectual stand fighting mathematics and *episteme* and trying to eradicate its foundation and kill it. Then, the tree that has killed its own roots will dry up and die; is anyone conscious of this simple relentless fact?

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